1.

i. Given that $y = e^{-x} \sin 2x$, find $\frac{dy}{dx}$.

[3]

ii. Hence show that the curve $y = e^{-x} \sin 2x$ has a stationary point when $x = \frac{1}{2}\arctan 2$.

[3]

2.

Given that $y = \ln\left(\sqrt{\frac{2x-1}{2x+1}}\right)$, show that $\frac{dy}{dx} = \frac{1}{2x-1} - \frac{1}{2x+1}$

[4]

3. Water flows into a bowl at a constant rate of 10 cm^3s^{-1} (see Fig. 4).

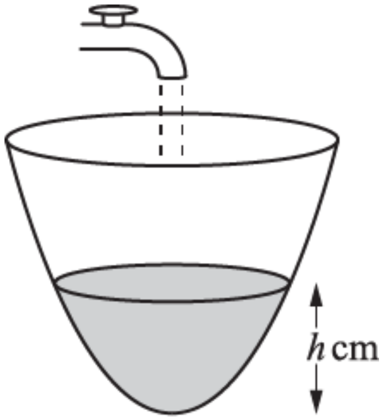

Fig. 4

When the depth of water in the bowl is h cm, the volume of water is V cm^3, where $V = \pi h^2$. Find the rate at which the depth is increasing at the instant in time when the depth is 5 cm.

[5]

4.

A spherical balloon of radius r cm has volume V cm^3, where $V = \frac{4}{3}\pi r^3$. The balloon is inflated at a constant rate of 10 cm^3 s^{-1}. Find the rate of increase of r when $r = 8$.

[5]

5.

Find the exact gradient of the curve $y = \ln(1 - \cos 2x)$ at the point with x-coordinat $\frac{1}{6}\pi$.

[5]

6. Fig. 4 shows a cone with its axis vertical. The angle between the axis and the slant edge is 45°. Water is poured into the cone at a constant rate of 5cm^3 per second. At time t seconds, the height of the water surface above the vertex O of the cone is h cm, and the volume of water in the cone is V cm^3.

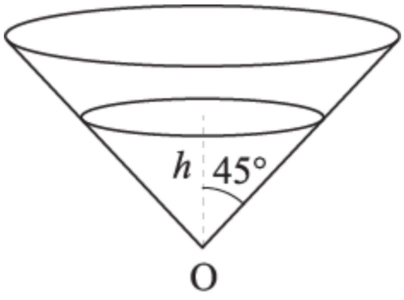

Fig. 4

Find V in terms of h.

Hence find the rate at which the height of water is increasing when the height is 10 cm.

[You are given that the volume V of a cone of height h and radius r is $V = \frac{1}{3}\pi r^2 h$].

[5]

7. Fig. 1 shows part of the curve $y = e^{2x} \cos x$.

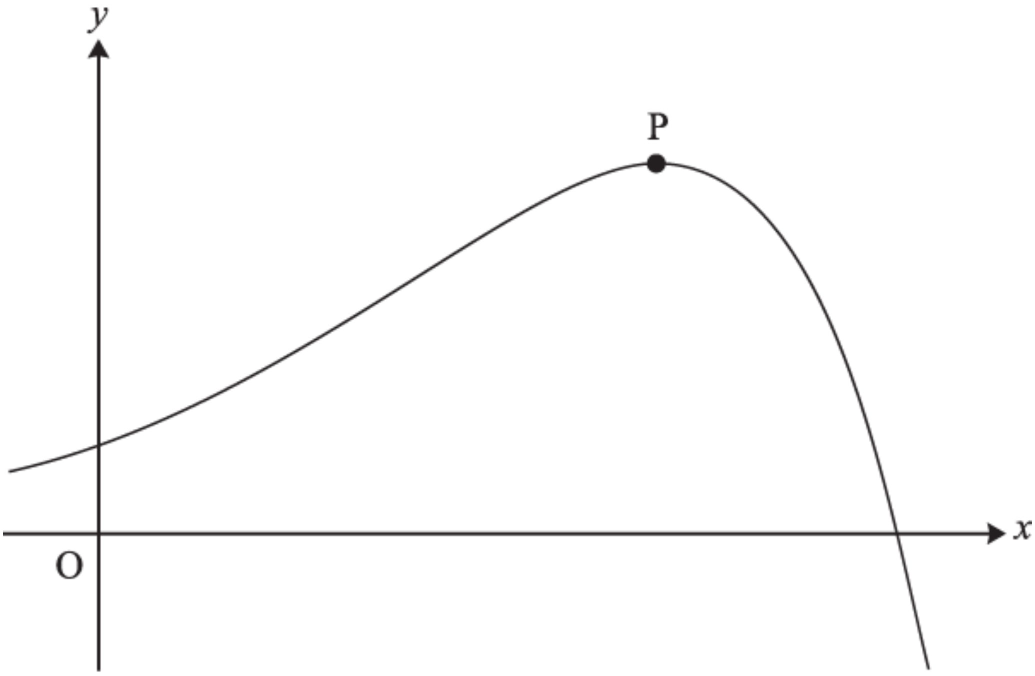

Fig. 1

Find the coordinates of the turning point P.

[6]

8. The volume V m^3 of a pile of grain of height h metres is modelled by the equation

$$V = 4\sqrt{h^3+1} - 4.$$

i. Find $\frac{dV}{dh}$ when $h = 2$.

[4]

At a certain time, the height of the pile is 2 metres, and grain is being added so that the volume is increasing at a rate of 0.4 m^3 per minute.

ii. Find the rate at which the height is increasing at this time.

[3]

9. Differentiate the following.

(a) $\sqrt{1-3x^2}$ [3]

(b) $\dfrac{x^2}{3x+2}$ [3]

10. The curve $y = \mathrm{f}(x)$ is defined by the function $\mathrm{f}(x) = \mathrm{e}^{-x} \sin x$ with domain $0 \leq x \leq 4\pi$.

(a)

(i) Show that the x-coordinates of the stationary points of the curve $y = \mathrm{f}(x)$, when arranged in increasing order, form an arithmetic sequence.

(ii) Show that the corresponding y-coordinates form a geometric sequence. [9]

(b) Would the result still hold with a larger domain? Give reasons for your answer [1]

11. Differentiate $(3x^2 + 5)^4$. [3]

12. Differentiate $\dfrac{1}{(5-2x^3)^2}$. [3]

13. Find $\frac{dy}{dx}$ given that $y = 3x^2 \sin 2x$. [3]

14. **In this question you must show detailed reasoning.**

A curve has equation $y = x - 5 + \frac{1}{x-2}$. The curve is shown in Fig. 4.

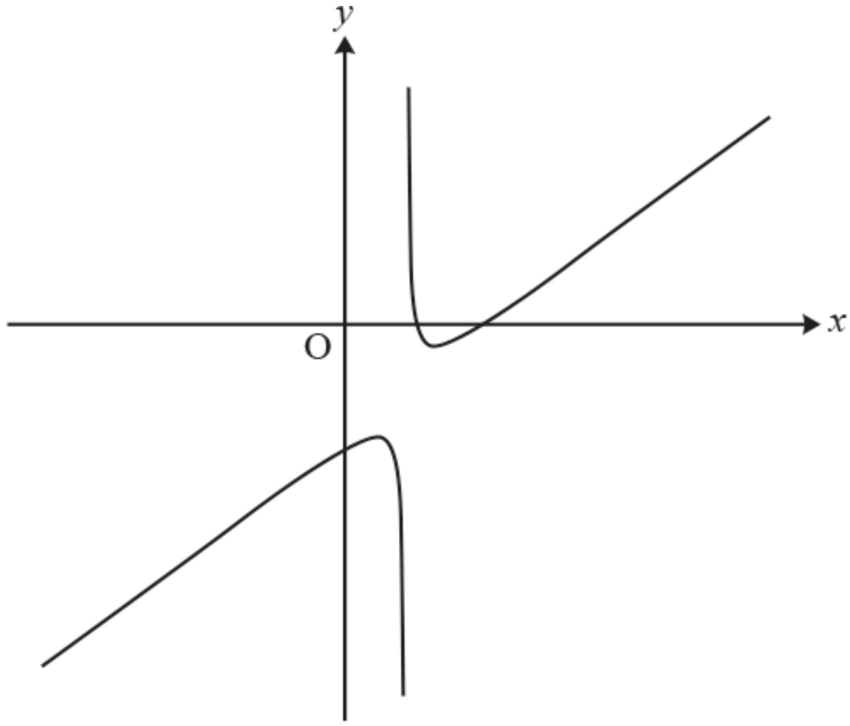

Fig. 4

(a) Determine the coordinates of the stationary points on the curve. [5]

(b) Determine the nature of each stationary point. [3]

(c) Write down the equation of the vertical asymptote. [1]

(d) Deduce the set of values of x for which the curve is concave upwards. [1]

15. You are given that

$$f(x) = x^4 - x,\ x > 1.$$

The graphs of $y = f(x)$, $y = x$ and $y = f^{-1}(x)$ are shown in Fig. 9.

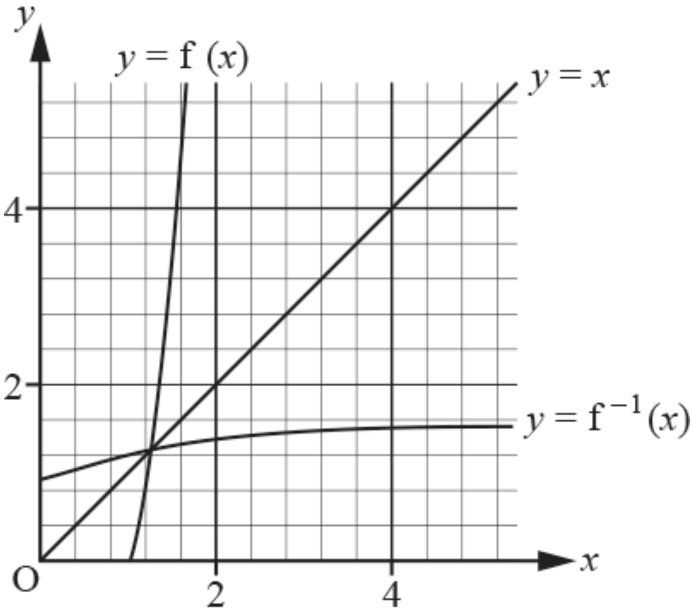

Fig. 9

(a) Show that the inverse function, $f^{-1}(x)$, passes through the point (14, 2). [1]

(b) Find the gradient of $f^{-1}(x)$ at the point (14, 2). [3]

16. **In this question you must show detailed reasoning.**

The equation of a curve is $y = \dfrac{\ln(x+2)}{\cos x}$.

Find the equation of the tangent to the curve at the point where $x = 0$. [5]

17. **In this question you must show detailed reasoning.**

(a) Find the exact coordinates of the stationary point of the curve $y = x \ln x$. [5]

(b) Show that the stationary point is a minimum turning point. [2]

18. The function $\mathrm{h}(x) = \sin\left(\frac{1}{x}\right)$ is defined for the domain $x > \frac{2}{\pi}$.

(a) Differentiate h(x) with respect to x. [2]

(b) Find the values between which $\frac{1}{x}$ lies for $x > \frac{2}{\pi}$. [2]

(c) Show that h(x) is a decreasing function. [3]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Part marks and guidance	
1		i	$y = e^{-x} \sin 2x$	M1	Product rule	$u \times$ their $v' + v \times$ their u'
		i	$\Rightarrow dy/dx = e^{-x}.2\cos 2x + (-e^{-x})\sin 2x$	B1	$d/dx(\sin 2x) = 2\cos 2x$ Any correct expression **Examiner's Comments**	
		i		A1	This proved to be a straightforward start to the paper, with the large majority of candidates getting full marks. Of those who did not, the most common errors were in the derivative of sin 2x (getting cos 2x or ½ cos 2x) or e-x (omitting the negative sign).	but mark final answer
		ii	$dy/dx = 0$ when $2\cos 2x - \sin 2x = 0$	M1	ft their dy/dx but must eliminate e^{-x}	derivative must have 2 terms
		ii	$' 2 = \tan 2x$	M1	$\sin 2x / \cos 2x = \tan 2x$ used	substituting ½ arctan 2 into their deriv M0
		ii	$' 2x = \arctan 2$		[or $\tan^{-1}$] NB AG **Examiner's Comments**	(unless $\cos 2x = 1/\sqrt{5}$ and $\sin 2x = 2/\sqrt{5}$ found)
		ii	$\Rightarrow x = ½ \arctan 2$*	A1	This part was somewhat less successful. Quite a few candidates just substituted the given answer into the derivative and claimed that this was zero.	must show previous step
			Total	6		

Question			Answer	Marks	Guidance	
2			$y=\ln\left(\sqrt{\dfrac{2x-1}{2x+1}}\right)=\dfrac{1}{2}(\ln(2x-1)-\ln(2x+1))$	M1	use of ln(*a*/*b*) = ln *a* – ln *b*	
				M1	use of ln√*c* = ½ ln *c*	
			$\Rightarrow \quad \dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{2}\left(\dfrac{2}{2x-1}-\dfrac{2}{2x+1}\right)$	A1	o.e.; correct expression (if this line of working is missing, M1M1A0A0)	
			$=\dfrac{1}{2x-1}-\dfrac{1}{2x+1}$ *	A1	NB AG for alternative methods, see additional solutions	
			Additional solutions			
			$y=\ln\left(\sqrt{\dfrac{2x-1}{2x+1}}\right)=\dfrac{1}{2}\ln\left(\dfrac{2x-1}{2x+1}\right)$	M1	ln √*c* = ½ ln *c* used	
			$\Rightarrow \dfrac{\mathrm{d}\,y}{\mathrm{d}\,x}=\dfrac{1}{2}\dfrac{1}{\left(\dfrac{2x-1}{2x+1}\right)}\dfrac{(2x+1)2-(2x-1)2}{(2x+1)^2}$			
			$=\dfrac{1}{2}\dfrac{2x+1}{2x-1}\dfrac{4}{(2x+1)^2}=\dfrac{2}{(2x-1)(2x+1)}$	A2	fully correct expression for the derivative	
			$\dfrac{1}{2x-1}-\dfrac{1}{2x+1}=\dfrac{2x+1-(2x-1)}{(2x-1)(2x+1)}$ $=\dfrac{2}{(2x-1)(2x+1)}$			
			$\dfrac{1}{2x-1}-\dfrac{1}{2x+1}=\dfrac{2x+1-(2x-1)}{(2x-1)(2x+1)}=\dfrac{2}{(2x-1)(2x+1)}$	A1	simplified and shown to be equivalent to $\dfrac{1}{2x-1}-\dfrac{1}{2x+1}$	

			Additional solutions			
			$y=\ln\left(\sqrt{\frac{2x-1}{2x+1}}\right)=\ln\sqrt{2x-1}-\ln\sqrt{2x+1}$	M1	ln(*a*/*b*) = ln *a* – ln *b* used	
			$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{2x-1}}\frac{1}{2}(2x-1)^{-1/2}.2-\frac{1}{\sqrt{2x+1}}\frac{1}{2}(2x+1)^{-1/2}$	A2	fully correct expression	
			$=\frac{1}{2x-1}-\frac{1}{2x+1}$	A1	simplified and shown to be equivalent to $\frac{1}{2x-1}-\frac{1}{2x+1}$	
			Additional solutions			
			$y=\ln\left(\sqrt{\frac{2x-1}{2x+1}}\right)$			
			$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\sqrt{\frac{2x-1}{2x+1}}}\frac{1}{2}\left(\frac{2x-1}{2x+1}\right)^{-1/2}\frac{(2x+1)2-(2x-1)2}{(2x+1)^2}$ or	M1	$\frac{1}{\boldsymbol{u}}$ × their *u'* where $\boldsymbol{u}=\sqrt{\frac{2x-1}{2x+1}}$ or $\frac{\sqrt{2x-1}}{\sqrt{2x+1}}$	
			$\frac{1}{\frac{\sqrt{2x-1}}{\sqrt{2x+1}}}\frac{\sqrt{2x+1}.\frac{1}{2}.2(2x-1)^{-1/2}-\sqrt{2x-1}.\frac{1}{2}.2(2x+1)^{-1/2}}{\sqrt{2x+1}^2}$	A2	(any attempt at *u'* will do) o.e. any completely correct expression for the derivative	
			$=\frac{1}{2}\left(\frac{2x+1}{2x-1}\right)\frac{4}{(2x+1)^2}=\frac{2}{(2x-1)(2x+1)}$		or $=\frac{\sqrt{2x+1}}{\sqrt{2x-1}}\frac{(2x+1)-(2x-1)}{(2x+1)^{3/2}(2x-1)^{1/}}$	

			$\frac{1}{2x-1}-\frac{1}{2x+1}=\frac{(2x+1)-(2x-1)}{(2x-1)(2x+1)}=\frac{2}{(2x-1)(2x+1)}$	A1	simplified and correctly shown to be equivalent to $\frac{1}{2x-1}-\frac{1}{2x+1}$ Examiner's Comments Some candidates spotted the trick of simplifying the given function to get $y = ½ \ln(2x - 1) - ½ \ln(2x + 1)$ before differentiating, and thereby made lives considerably easier for themselves! However, writing the answer down from here omitted the vital 2 x ½ working and lost two marks. Those who started differentiating from $y = \ln(\sqrt{(2x-1)}) - \ln(\sqrt{(2x+1)})$ needed to convince that they were using a chain rule on $\sqrt{u}$, where $u = 2x - 1$. Some tenacious candidates even managed to differentiate the given function correctly without these preliminaries, but made life hard for themselves.	
			Total	4		
3			$V = \pi h^2 \Rightarrow \mathrm{d}V/\mathrm{d}h = 2\pi h \Rightarrow$	M1A1	if derivative $2\pi h$ seen without $\mathrm{d}V/\mathrm{d}h = \ldots$ allow M1A0	
			$\mathrm{d}V/\mathrm{d}t = \mathrm{d}V/\mathrm{d}h \times \mathrm{d}h/\mathrm{d}t$	M1	soi; o.e. – any correct statement of the chain rule using V, h and t – condone use of a letter other than t for time here	
			$\mathrm{d}V/\mathrm{d}t = 10$	B1	soi; if a letter other than t used (and not defined) B0	
			$\mathrm{d}h/\mathrm{d}t = 10/(2\pi \times 5) = 1/\pi$	A1	or 0.32 or better, mark final answer Examiner's Comments This proved to be an accessible 5 marks, with many candidates getting the question fully correct. Of those who did not, $\mathrm{d}h/\mathrm{d}t = 10$ (instead of $\mathrm{d}V/\mathrm{d}t$) was quite a common misconception; some tried to find $\mathrm{d}h/\mathrm{d}V$ but failed to handle the constant of $1/\sqrt{\pi}$ correctly; and a surprising number finished off by saying that $10/10\pi = \pi$ instead of $1/\pi$.	

			Total	5		Product, Quotient and Chain Rules
4			$dV/dr = 4\pi r^2$	B1	or $12\pi r^2/3$, condone dr/dV, dV/dR	
			$dV/dt = 10$	B1		Condone use of other letters for t
			$dV/dt = (dV/dr)(dr/dt)$	M1	a correct chain rule soi	o.e. e.g. $dr/dt = (dr/dV)(dV/dt)$
			$\Rightarrow 10 = 4\pi.64.dr/dt$	A1	o.e. (soi) must be correct	
					0.012 or better or $10/256\pi$ or $5/128\pi$	
					Examiner's Comments	
			$\Rightarrow dr/dt = 0.0124$ cm s^{-1}	A1	This question proved to be accessible to the overwhelming majority of candidates, and there were many fully correct solutions. Even those who failed to get full marks usually picked up an M1 for a correctly stated chain rule, B1 for $dV/dr = 4\pi r^2$, and a B1 for $dV/dt = 10$. Approximate answers are perhaps preferable in a contextual question, but exact answers were also allowed.	mark final answer
			Total	5		
5			$y = \ln(1 - \cos 2x)$, let $u = 1 - \cos 2x$			
			$\Rightarrow dy/dx = dy/du.\ du/dx$	M1	$1/(1 - \cos 2x)$soi	
			$= (1/u).\ 2\sin 2x$	M1	$d/dx(1 - \cos 2x) = \pm 2\sin 2x$	
			$= \dfrac{2\sin 2x}{1-\cos 2x}$	A1cao		
			When $x = \pi/6$, $\dfrac{dy}{dx} = \dfrac{2\sin(\pi/3)}{1-\cos(\pi/3)}$	M1	substituting $\pi/6$ or 30° into their derive	must be in at least two places

			$= 2\sqrt{3}$	A1cao	Examiner's Comments Plenty of candidates scored 5 marks here with little difficulty. Some missed out the derivative of 1–cos2x, and some wrote 1/2sin2x instead of 1/(1-cos2x). The substitution of √/6 into the correct derivative was usually done correctly. Some approximation of 2√3 was found, but could usually be condoned by ignoring subsequent working.	isw after correct answer seen
			Total	5		
6			$h = r$ so $V = \pi h^3/3$	B1	o.e. e.g $\pi h^3 \tan 45°/3$	
			$\mathrm{d}V/\mathrm{d}t = 5$	B1	soi (can be implied from $V = 5t$)	e.g. from a correct chain rule
			$\mathrm{d}V/\mathrm{d}h = \pi h^2$	B1ft	must be $\mathrm{d}V/\mathrm{d}h$ soi, ft their $\pi h^3/3$	but must have substituted for r
			$\mathrm{d}V/\mathrm{d}t = (\mathrm{d}V/\mathrm{d}h).\mathrm{d}h/\mathrm{d}t$	M1	any correct chain rule in V, h and t (soi)	e.g. $\mathrm{d}h/\mathrm{d}t = \mathrm{d}h/\mathrm{d}V \times \mathrm{d}V/\mathrm{d}t$,
			$\Rightarrow 5 = 100\pi\, \mathrm{d}h/\mathrm{d}t$			
			$\Rightarrow \mathrm{d}h/\mathrm{d}t = 5/100\pi = 0.016$ cm s^{-1}	A1	0.016 or better; accept 1/(20 π) o.e., but mark final answer	0.01591549 … penalise incorrect rounding penalise incorrect rounding
			or $V = 5t$ so $\pi h^3/3 = 5t$	B1		
			$\Rightarrow \pi h^2 \mathrm{d}h/\mathrm{d}t = 5$	M1	or $5\, \mathrm{d}t/\mathrm{d}h = \pi h^2$ o.e.	
					0.016 or better; accept 1/(20 π) o.e., but mark final answer	
			$\Rightarrow \mathrm{d}h/\mathrm{d}t = 5/\pi h^2 = 5/100\pi = 0.016$ cm s^{-1}	A1	Examiner's Comments This question was less well done. Nearly all candidates gained marks for quoting a correct chain rule and using dV/dt = 5. By far the most common error thereafter was to fail to find V as a function of h and instead differentiating V = $\pi r^2 h/3$ to give dV/dh = $\pi r^2/3$. Even when candidates	Penalise incorrect rounding

					recognised the need to substitute for r, there were a surprising number of trigonometric errors, such as h = r sin 45°. A number of solutions which found dh/dt = 1/20π; then went on to write or evaluate this as π/20.	
			Total	5		
7			$y = e^{2x}\cos x$	M1	product rule used	consistent with their derivs
			$\Rightarrow dy/dx = 2e^{2x}\cos x - e^{2x}\sin x$	A1	cao – mark final ans	e.g. $2e^{2x} - e^{2x}\tan x$ is A0
			$dy/dx = 0 \Rightarrow e^{2x}(2\cos x - \sin x) = 0$	M1	their derivative = 0	
			$\Rightarrow 2\cos x = \sin x$			
			$\Rightarrow 2 = \sin x/\cos x = \tan x$	M1	$\sin x/\cos x = \tan x$ used	or $\sin^2 x + \cos^2 x = 1$ used
			$\Rightarrow x = 1.11$	A1	1.1 or 0.35 π or better, or arctan 2, not 63.4° but condone ans given in both degrees and radians here	1.1071487 …, 0.352416 … π, penalise incorrect rounding
			$\Rightarrow y = 4.09$	A1cao	art 4.1 **Examiner's Comments** The product rule was done well, and most candidates were successful in arriving at tanx = 2 at the turning point. The most common error was to give x in degrees and then touse this to calculate y, giving a rather alarmingly large result!	no choice
			Total	6		
8		i	$dV/dh = 4.½\,(h^3 + 1)^{-1/2}\,.3h^2$	M1	chain rule	their deriv of $4u^{1/2}$ × their deriv of $h^3 + 1$
		i		A1	correct	
		i		M1	substituting $h = 2$ into their derivative	
		i	when $h = 2$, $dV/dh = 8$	A1cao		

						Examiner's Comments This question was extremely well answered, with the majority of candidates scoring full marks. The chain rule on V was successfully negotiated by over half the candidates, and then correctly evaluated at $x = 2$.
		ii	$\mathrm{d}V/\mathrm{d}t = 0.4$	B1	soi	condone r for t
		ii	$\mathrm{d}V/\mathrm{d}t = \mathrm{d}V/\mathrm{d}h \times \mathrm{d}h/\mathrm{d}t$	M1	o.e.	any correct chain rule in V, h, t (or r)
		ii	$0.4 = 8 \times \mathrm{d}h/\mathrm{d}t \Rightarrow \mathrm{d}h/\mathrm{d}t = 0.05$ (m per min)	A1cao	0.05 or 1/20	**Examiner's Comments** This question was extremely well answered, with the majority of candidates scoring full marks. Virtually everyone who scored 4 for part (i) went on to apply the chain rule $\mathrm{d}V/\mathrm{d}t = \mathrm{d}V/\mathrm{d}h \times \mathrm{d}h/\mathrm{d}t$, or some variation of it, to get full marks here. The rest usually earned the first two of the three marks.
			Total	7		
9		a	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}\left(1 - 3x^2\right)^{-\frac{1}{2}}.(-6x)$	B1(AO1.1) M1(AO1.1) A1(AO1.1)	$\frac{1}{2}u^{-\frac{1}{2}}$ soi Chain rule	

			$= \dfrac{-3x}{\sqrt{(1-3x^2)}}$	[3]	oe, but must simplify $\dfrac{1}{2}\times 6$	
		b	$\dfrac{dy}{dx}=\dfrac{(3x+2).2x-x^2.3}{(3x+2)^2}$ $=\dfrac{3x^2+4x}{(3x+2)^2}$	M1(AO1.1) A1(AO1.1) A1(AO1.1) [3]	Quotient rule or product rule oe, but must simplify numerator	
			Total	6		
10		a	(i) $f'(x) = e^{-x}\cos x - e^{-x}\sin x$ $f'(x) = 0$ and $e^{-x} \neq 0 \Rightarrow \cos x = \sin x$ $\Rightarrow \tan x = 1$ $\Rightarrow x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}, \dfrac{13\pi}{4}$	M1(AO3.1a) A1(AO1.1) E1(AO2.2a) M1(AO1.1) A1(AO1.1)	product rule correct Use of $\dfrac{\sin}{\cos} = \tan$ $x = \dfrac{\pi}{4}$ (condone 45°) $\dfrac{\pi}{4}, \dfrac{5\pi}{4}, \dfrac{9\pi}{4}, \ldots$	
		a	So an AP with $d = \pi$ (ii) $y=\dfrac{\sqrt{2}}{2}e^{-\frac{\pi}{4}}, -\dfrac{\sqrt{2}}{2}e^{-\frac{5\pi}{4}}, \dfrac{\sqrt{2}}{2}e^{-\frac{9\pi}{4}}, -\dfrac{\sqrt{2}}{2}e^{-\frac{13\pi}{4}}$	E1FT(AO2.1) M1(AO3.1a) A1(AO1.1)	must state the common difference	FT their values of x

			This is a GP with $r = -e^{-\pi}$	E1FT(AO2.1) [9]	substituting one value of x into f(x) must state common ratio, www	FT their values of y
		b	Yes with explanation that values of x would continue to be separated by pi and so values of y would continue to have same common ratio.	E1(AO2.2a) [1]		
			Total	10		
11			$\frac{dy}{dx} = 4(3x^2+5)^3 \times 6x$ $\frac{dy}{dx} = 24x(3x^2+5)^3$	M1(AO1.1a) A1(AO 1.1b) A1(AO 1.1b) [3]	Use of chain rule attempted 6x soi	
			Total	3		
12			$\frac{1}{(5-2x^3)^2} = (5-2x^3)^{-2}$ $\frac{d}{dx}(5-2x^3)^{-2} = (-6x^2)(-2)(5-2x^3)^{-3}$	M1	Chain rule	or quotient (or

			$= 12x^2 (5 - 2x^3)^{-3}$ isw	A1 A1cao [3]	on $(5 - 2x^3)^{-2}$ correct expression, allow $(-6x^2)(-2)u^{-3}$ o.e. or $\frac{12x^2}{(5-2x^3)^3}$ isw | product) e.g. $\frac{(5-2x^3)^2.0 - 1.2(5-2x^3)(-6x^2)}{(5-2x^3)^4}$ M1A1 [must have correct denom for M1] $u\,v' - v\,u'$ in QR is M0 Examiner's Comments This was a straightforward test of the chain rule, in which over three quarters of the candidates scored full marks. Occasionally we saw a quotient rule used, which required to be simplified to gain full marks. Another occasional error was to get the wrong sign, e.g. $-12x^2(5 - 2x^3)^{-3}$.	
			Total	3		
13			Product rule with $u = 3x^2$ and $v = \sin 2x$ $\frac{dy}{dx} = 6x\sin 2x + 6x^2\cos 2x$	M1(AO1.1a) A1(AO1.1b) A1(AO1.1b) [3]	Need not be written explicitly For either of the two terms correct For completely correct answer	
			Total	3		

14	a	$\frac{dy}{dx} = 1 - \frac{1}{(x-2)^2}$ $1 - \frac{1}{(x-2)^2} = 0$ at stationary points $x - 2 = \pm 1$ so $x = 1, 3$ $(1, -5)$ $(3, -1)$	M1(AO 1.1a) A1(AO 1.1) M1(AO 1.1a) A1(AO 2.2a) A1(AO 1.1) [5]	Attempt to differentiate with one term correct Correct derivative Both values of *x* Both values of *y* – ft *their x* **Examiner's Comments** Most candidates were able to score full marks here following correct differentiation and solution of what ended up as a quadratic equation. Many different ways were used to solve the equation usually without any wrong working. Examiners were pleased to see correct notation used in this question.
	b	$\frac{d^2y}{dx^2} = \frac{2}{(x-2)^3}$ $x = 3$ $\frac{d^2y}{dx^2} > 0$ (2) so minimum	M1(AO 1.1a) A1(AO 2.4) A1(AO 2.4)	

			$x=1 \quad \dfrac{d^2y}{dx^2}<0$	(–2) so maximum	[3]	OR Allow consideration of gradient either side of stationary point for **M1** Correct gradients above and below each tp **A1** Correct convincing conclusions (possibly with sketches) **A1**	

x	$f'(x)$
0.5	0.56
0.6	0.49
0.7	0.41
0.8	0.31
0.9	0.17
1	0.00
1.1	-0.23
1.2	-0.56
1.3	-1.04
1.4	-1.78
1.5	-3.00

					Examiner's Comments Again most candidates were successful in classifying the stationary points with use of the second derivative being the most common method. A few considered the gradient either side of each turning point and then reasoned their way to a correct conclusion.	
		c	$x = 2$	B1(AO 1.2) [1]	**Examiner's Comments** There appeared to be confusion as to the meaning of vertical asymptote which led to a low success rate for this part.	
		d	$x > 2$	A1(AO 2.2a) [1]	FT *their* (c) if region is to right of *their* x value **Examiner's Comments** As in part (c), a large proportion of the candidates struggled with this part. AfL The OCR B (MEI) H640 specification defines the terms "concave upwards" and "concave downwards" as those that will be used in examination questions.	

			Total	10		
15		a	$f(2) = 16 - 2 = 14$. Since $f(x)$ passes through 2, 14), $f^{-1}(x)$ must pass through (14, 2)	E1 (AO2.4) [1]		
		b	$f'(x) = 4x^3 - 1$ $f'(2) = 31$ $\frac{1}{31}$	B1 (AO2.1) M1 (AO1.1) A1 (AO1.1) [3]	Accept BC	
			Total	4		
16			DR $$\frac{dy}{dx} = \frac{\cos x \times \frac{1}{x+2} - \ln(x+2)(-\sin x)}{(\cos x)^2}$$ $x = 0$ $y = \ln 2$ $$x = 0, \frac{dy}{dx} = \frac{1}{2}$$ So equation of tangent is $y = ½x + \ln 2$	M1(AO 3.1a) A1(AO 1.1) B1(AO 1.1) M1(AO 1.1) A1(AO 1.1) [5]	Use of quotient rule	
			Total	5		
17		a	DR $$\frac{dy}{dx} = 1 + \ln x$$	B1(AO 1.1a) B1(AO 1.1)	B1 for each term	

			At stationary point ln $x = -1$ $x = \frac{1}{e}$ $y = -\frac{1}{e}$	M1(AO 1.1) A1(AO 2.2a) A1(AO 2.2a) [5]	correct $\left(\frac{1}{e}, -\frac{1}{e}\right)$	
		b	DR $\frac{d^2y}{dx^2} = \frac{1}{x}$ At stationary point $\frac{d^2y}{dx^2} = e$ positive so minimum	M1(AO 1.1a) E1(AO 2.4) [2]	**AG** Convincing conclusion needed	
			Total	7		
18		a	$-\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$	B2 (AO 1.1a, 1.1) [1]	**B1** for $-\frac{1}{x^2}$ or $\cos\left(\frac{1}{x}\right)$	

		b	$\frac{1}{x}>0$ $\frac{1}{x}<\frac{\pi}{2}$	B1 (AO 2.2a) B1 (AO 2.2a) [2]	Correct lower limit 0 stated Correct upper limit $\frac{\pi}{2}$ stated	
		c	$0<\frac{1}{x}<\frac{\pi}{2}\Rightarrow\cos\left(\frac{1}{x}\right)>0$ $-\frac{1}{x^2}<0$ Hence h′(x) < 0 so h (x) is a decreasing function	M1 (AO 2.2a) M1 (AO 1.1) A1 (AO 2.4) [3]	**AG** Convincing completion needed	
			Total	7		